UUCMS NO

B.M.S COLLEGE FOR WOMEN AUTONOMOUS

BENGALURU -560004

END SEMESTER EXAMINATION – APRIL/ MAY 2023

M.Sc. Mathematics – I Semester

DISCRETE MATHEMATICS

Course Code MM105T **QP Code: 11005**

Duration: 3 Hours **Maximum Marks: 70**

Instructions: 1) **All** questions carry **equal** marks.
2) Answer **any five** full questions.

1. a) Test the validity of the following argument
"If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. I will not be happy. Therefore, either I will not get the job or I will not work hard."
b) Using the method of proof by contradiction prove that $\sqrt{2}$ is irrational.
c) Obtain the principal disjunctive normal form of $(\neg p \to r) \wedge (q \leftrightarrow p)$ without using truth table. **(5+4+5)**

2. a) A computer company receives 350 applications from computer graduates for a job planning a line of new web servers. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?
b) How many ways are there to arrange the letters in the word (i) SYSTEMS (ii) BANANA ?
c) Show that if any 11 numbers are chosen from the set {1, 2… 20} then one of them will be a multiple of another. **(4+5+5)**

3. a) Model the tower of Hanoi puzzle as recurrence relation and solve it explicitly.
b) Solve the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ for $n \geq 3$ with initial conditions $a_0 = 3$, $a_1 = 5$ and $a_2 = 15$.
c) Using generating functions solve the recurrence relation $a_n = 3a_{n-1} + 2$ with $a_0 = 1$. **(5+5+4)**

4. a) Let A be a set with n elements and R be a relation on A. Let M be the matrix of R then prove that

   (i) R is reflexive if and only if $I_n \leq M$, where I_n is the identity matrix of order n.

   (ii) R is antisymmetric if and only if $M \cap M^T \leq I_n$.

   (iii) R is transitive if and only if $M^2 \leq M$.

   b) Using Warshall's algorithm find the transitive closure R^* of the relation $R = \{(1,2),(1,5),(2,1),(2,3),(3,4),(4,5),(5,2),(5,3)\}$ on $A = \{1,2,3,4,5\}$.

   c) Draw the Hasse diagram of $(D_{81}, |)$.

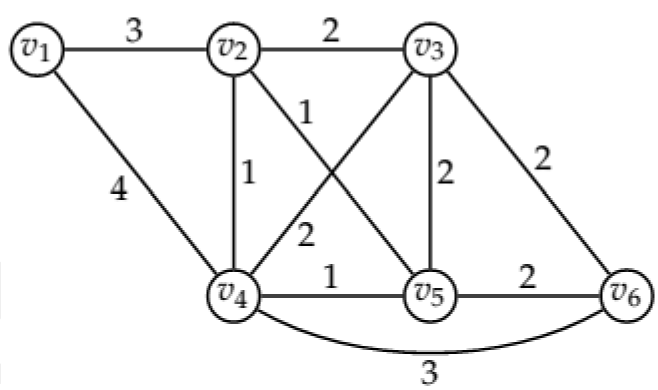
(5+5+4)

5. a) Prove that in a simple connected graph G, every $u - v$ walk contains a u – v path.

   b) Define a self – complementary graph $\overline{\text{G}}$ of a graph G and give an example. Show that every self – complementary graph has $4n$ or $4n + 1$ vertices, where n is a positive integer.

   c) If $G = (V, E)$ is a (p, q) graph, then show that $\delta(\text{G}) \leq \frac{2\text{q}}{\text{p}} \leq \Delta(\text{G})$, where $\delta(G)$ and $\Delta(\text{G})$ denote the minimum and maximum degree of G respectively. **(5+6+3)**

6. a) For the following weighted graph find the minimum spanning tree using Kruskal's algorithm.

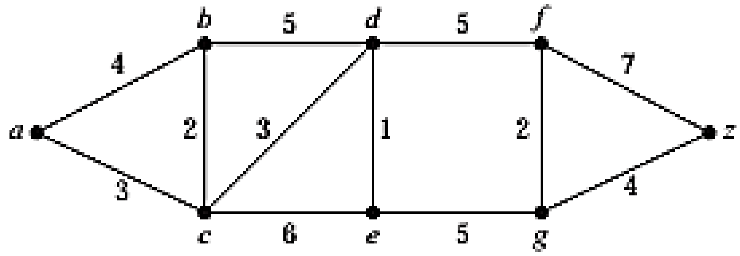


   b) Prove that the number of pendent vertices in a binary tree with p vertices is $\left(\frac{p+1}{2}\right)$.

   c) Prove that a tree with p vertices has $(p-1)$ edges. **(6+5+4)**

7. a) Prove that in a graph G, $k(G) \leq \lambda(G) \leq \delta(G)$, with standard notations.

   b) Using Dijkstra's algorithm find the shortest path from 'a' to all other vertices.



c) Prove that a vertex v is a cut-vertex in a graph G if and only if there exist two vertices u and w in G such that v lies on every $u - w$ path. **(5+5+4)**

8. a) Prove that connected graph G is Eulerian if and only if all vertices of G are even.
   b) Define a Hamiltonian graph and give an example. Let G be a simple graph with $p \geq 3$ vertices. If $\deg(u) + \deg(v) \geq n$ for every pair of non – adjacent vertices u and v of G, then G is Hamiltonian.
   c) Define a planar graph. If G is a simple connected planar graph with 'p' vertices and $q > 1$ edges then show that $q \leq 3p - 6$. Test the planarity of K_5. **(5+5+4)**